1. (20pts) A mass of \( R(1) \) kg moves on a circular spiral currently of radius \( R(2) \). It is know that the radius is increasing at a rate of \( R(1)R(2)/10 \) m/s and radially accelerating at \( R(1)R(2)/5 \) m/s\(^2\). It is known that the angle of the radius vector to the mass is \( \theta = R(3)R(4) \) degrees and the angle is advancing at a rate of \( R(5) \) rad/s and it is decelerating at \( R(6) \) rad/s\(^2\). This entire spiral system is rotating about an axes out of the plane of the spiral by a constant amount 3 rad/s. Compute the force the mass experiences at the instant of observation. \( R(1) \) is the first nonzero digit of your student R-number, \( R(2) \) is the second nonzero digit, etc. So if your R-number is \( R = 0234607 \), then \( R(1) = 2 \) and \( R(2)R(3) = 34 \), etc. Ignore the force from gravity.

\[
\vec{F} = m\hat{r} - p
\]

\[
\vec{u}_r = r \hat{r}
\]

\[
\vec{u}_\theta = \hat{\theta} + r(\dot{\theta} + \omega)\hat{b}_2
\]

\[
\vec{u}_\phi = \hat{\phi} + r(\dot{\phi} + \omega)\hat{b}_2 + \hat{\theta} \times (\hat{\phi} \times (\hat{r} + r(\dot{\phi} + \omega)\hat{b}_2))
\]

\[
= (\ddot{r} - r(\dot{\theta} + \omega)^2)\hat{r} + (2\dot{r}(\dot{\phi} + \omega) + r\phi)\hat{b}_2
\]

\[
\vec{F} = m\left[ (\ddot{r} - r(\dot{\theta} + \omega)^2)\hat{r} + (2\dot{r}(\dot{\phi} + \omega) + r\dot{\phi})\hat{b}_2 \right]
\]

\[
m = R(1), \quad \dot{r} = R(2), \quad \ddot{r} = \frac{R(1)R(2)}{10}, \quad \dot{\phi} = \frac{R(1)R(2)}{5}, \quad \dot{\theta} = R(5), \quad \ddot{\theta} = R(6)
\]

\[
\omega = 3
\]
2. (20pts) At the instant $\theta = \frac{\pi}{2}$ find $\dot{\beta}$, $\ddot{\beta}$, and $\dddot{\beta}$. Let $\theta(t) = R(1)t^2$ where $R(1)$ carries units $\text{rad}/\text{s}^3$, $B = R(2)$ ft, and $C = 2B$ ft. $R(1)$ is the first nonzero digit of your student R-number, $R(2)$ is the second nonzero digit, etc. So if your R-number is $R = 0234607$, then $R(1) = 2$ and $R(2)R(3) = 34$, etc.

\[ \theta = R(1)t^2 \Rightarrow \dot{\theta} = 2R(1)t, \quad \ddot{\theta} = 2R(1) \]
\[ \dot{\theta} = 2R(1) \sqrt{\frac{1}{B}}, \quad \ddot{\theta} = 2R(1) \]
\[ \dot{\theta} = 2R(1) \sqrt{\frac{1}{2R(1)}} \]
\[ \dddot{\theta} = \sqrt{2}\pi R(1) \]

**Quickest Method.**

\[ \frac{\sin \beta}{B} = \frac{\sin \theta}{C} \]
\[ \sin \beta = \frac{B}{C} \sin \theta \Rightarrow \beta = \sin^{-1}\left(\frac{B}{C} \sin \theta\right) \]
\[ \beta = \sin^{-1}\left(\frac{B}{C} \right) \]
\[ C = 2B \]
\[ \beta = \sin^{-1}\left(\frac{B}{C} \right) \]
\[ \beta = R(2) \]

**Image P. differential.**

\[ \dot{\beta} = \frac{B}{C} \dot{\theta} \cos \beta \Rightarrow \dot{\beta} = \frac{B}{C} \dot{\theta} \cos \beta \]
\[ \dot{\beta} \left(\cos \beta - (90^\circ) \right) = \frac{B}{C} \dot{\theta} \cos \beta - \dot{\theta}^2 \sin \beta \]

\[ \dot{\beta} = \frac{\dot{\theta}^2}{\cos \beta} \]
\[ B = 30^\circ \]

\[ \dot{\beta} = \frac{\dot{\theta}^2}{\cos \beta} = \frac{\dot{\theta}^2}{\cos (30^\circ)} = \frac{\dot{\theta}^2}{\frac{\sqrt{3}}{2}} = \frac{\dot{\theta}^2}{\sqrt{3}} \]
3. (60 pts) The piston cylinder arrangement \( o-b-c \) is followed by the slotted bar \( a-p \) connected at sliding pin \( c \). The crank at \( o \) is driven such that \( \theta \) is known as a function of time. Find:

(a) the transformation expressions relating frames \( A, B \) and \( C \), local vectors to the \( N \) vectors, use positive right-hand sense for the frames as drawn,

(b) the number of degrees of freedom and the vector loop expression(s) that exist for this device in terms of local vectors and base \( N \) frame vectors, but contain no explicit trigonometric functions; in other words write them in terms of local vectors as needed,

(c) expressions relating \( \dot{\beta}, \dot{x}, \dot{y}, \) and \( \dot{\phi} \) to \( \dot{\theta} \) in terms of unresolved vector dot products,

(d) expressions relating \( \ddot{\beta}, \ddot{x}, \ddot{y}, \) and \( \ddot{\phi} \) to \( \ddot{\theta} \) and \( \ddot{\theta} \) in terms of unresolved vector dot products,

(e) the expression for the absolute velocity of points \( b, c \) and \( p \) but using local frame vectors, symbols representing items found above need not be replaced and can be left as symbols, and

(f) the expression for the absolute acceleration of points \( b, c \) and \( p \) but using local frame vectors, symbols representing items found above need not be replaced and can be left as symbols.

This would be mostly be step 5 of the 6 step process.

\[
\begin{align*}
\begin{pmatrix}
\mathbf{w}_A \\
\mathbf{w}_B \\
\mathbf{w}_C
\end{pmatrix} &=
\begin{pmatrix}
\dot{\alpha}_3 \\
\dot{\beta}_3 \\
\dot{\gamma}_3
\end{pmatrix}
\end{align*}
\]

\[
\begin{align*}
\mathbf{a}_1 &= -\omega_2 \psi \mathbf{\hat{n}}_1 + \sin \psi \mathbf{\hat{n}}_1 \\
\mathbf{\hat{n}}_1 &= \sin \psi \mathbf{\hat{n}}_2 + \cos \psi \mathbf{\hat{n}}_1 \\
\mathbf{\hat{n}}_2 &= \mathbf{\hat{n}}_2 \\
\mathbf{\hat{n}}_3 &= \omega_3 \mathbf{\hat{n}}_1 + \sin \theta \mathbf{\hat{n}}_2 \\
\mathbf{\hat{n}}_4 &= -\sin \theta \mathbf{\hat{n}}_1 + \cos \theta \mathbf{\hat{n}}_2 \\
\mathbf{\hat{n}}_3 &= \mathbf{\hat{n}}_3 \\
\mathbf{\hat{n}}_4 &= -\omega_3 \mathbf{\hat{n}}_1 + \sin \beta \mathbf{\hat{n}}_2 \\
\mathbf{\hat{n}}_5 &= \sin \beta \mathbf{\hat{n}}_1 + \cos \beta \mathbf{\hat{n}}_2 \\
\mathbf{\hat{n}}_3 &= -\mathbf{\hat{n}}_3
\end{align*}
\]

\[
\begin{align*}
0 &= \mathbf{r}_b \mathbf{\dot{r}}_b + \mathbf{r}_c \mathbf{\dot{r}}_c + \mathbf{r}_o \mathbf{\dot{r}}_o \\
\mathbf{v}_{\text{tip}} &= \mathbf{H} \mathbf{\dot{n}}_2 + \mathbf{y} \mathbf{\dot{n}}_1 - \mathbf{x} \mathbf{\dot{n}}_1 = 0
\end{align*}
\]
\[ V_{\text{loop 1}} = -B_1 \beta \hat{\theta} \hat{\phi}^2 - B_2 \beta \hat{\theta}^2 \hat{\phi} - \left( B_3 \beta \hat{\theta} - C \beta^2 \hat{\phi} \right) - \chi \hat{\phi} = 0 \]

\[ \dot{\phi} = \frac{B_1 \beta \hat{\theta} \hat{\phi}^2 + B_2 \beta \hat{\theta}^2 \hat{\phi} - \chi \hat{\phi}}{C \beta \hat{\phi}^2} \]

\[ V_{\text{loop 2}} = (y - y\phi^2) \hat{\theta} + (2y\dot{\phi} + y\phi^2) \hat{\phi} - \chi \hat{\phi} = 0 \]

\[ \dot{\theta} = (B_1 \beta \hat{\theta} \hat{\phi}^2 - B_2 \beta \hat{\theta}^2 \hat{\phi} - C \beta^2 \hat{\phi} \hat{\theta} - C \beta^2 \hat{\phi} \hat{\theta}^2) / C \beta \hat{\phi}^2 \]

\[ \dot{x} = x(\hat{\phi}, \hat{\theta}) \]

\[ \dot{y} = x(\hat{\phi}, \hat{\theta}) - y(\hat{\phi}, \hat{\theta}) \]

\[ \dot{\phi} = \frac{x(\hat{\phi}, \hat{\theta}) - 2y \phi}{y} \]

\[ \dot{\phi} = B_1 \beta \hat{\theta} \hat{\phi} + B_2 \beta \hat{\theta}^2 \hat{\phi} - \chi \hat{\phi} = \dot{\phi} \hat{\phi} \]

\[ \dot{\phi} = B_1 \beta \hat{\theta} - B_2 \beta^2 \hat{\phi} - \chi \hat{\phi} = \dot{\phi} \hat{\phi} - \dot{A} \phi \hat{\phi} \]