Quiz 12 Solutions

Peter McDonough

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ME2302: Quiz # 12 Takehome

Due in Dr. Barhorst’s mailbox COB Monday 05/05/14. Work alone!

1. A toy marble track has its starting point $H$ units above the ground. The track slopes down at 45 degrees from the horizontal and turns within a circular arc of radius $R$ that is tangent to the ground then the track follows a 50 degree slope after the arc and terminates at height $h < H$. A marble of mass $m$, radius $r < R$, is released from rest at the start of the track. Treating the marble as a point mass, find the following:

(a) the acceleration of the marble at the bottom of the circular arc,
(b) the velocity vector of the marble as it leaves the track and becomes an assumed frictionless projectile,
(c) the acceleration of the marble as it peaks in its free flight, and
(d) the distance it travels away from the point it was launched as it hits the ground the first time.

2. Repeat problem 1 but treat the marble as a sphere and assume it rolls without slip on the track.

3. Regarding problems 1 and 2, if the coefficient of restitution is $e_r$ for impact with the ground, and it is assumed the impact with the ground completely stops the marble spin, which situation of the two produces the greater distance before the second bounce, compute those distances. Hint pay attention to angular as well as linear momentum at the first bounce.

4. A trucking company wants to have enough power to carry a total 50 US Ton truck and payload up a 2 mile long 15 degree grade at an average of 25 mph. Don’t ignore air drag, Assume the truck has a frontal area of 9 yards square, and a Coefficient of drag $C_d = 0.8$. How much power should the engine be capable of producing?

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**Problem 1**

This is supposed to be a circular track. This is a terrible sketch.
Part A

Conserve energy from top of hill to bottom of circular arc, then use the single rotated frame formulas for acceleration.

\[
\begin{align*}
\text{In}[1]:= & \quad \text{Vi} = m g H; \\
& \text{Ti} = 0; \\
& \text{Vf} = 0; \\
& \text{Tf} = \frac{1}{2} m v^2; \\
& \text{eq} = \text{Vi} + \text{Ti} = \text{Vf} + \text{Tf}
\end{align*}
\]

\[
\text{Out}[5] := g H m = \frac{m v^2}{2}
\]

\[
\text{In}[6] := \quad \text{Solve[eq, v]} \quad \text{// Last}
\]

\[
\text{Out}[6] := \left\{ v \rightarrow \sqrt{2} \sqrt{g H} \right\}
\]

The acceleration of the marble at the bottom of the arc is

\[
\begin{align*}
\text{In}[7]:= & \quad \text{acci} = \left\{-r''[t] + \vartheta''[t]^2 r[t], \ 2 \vartheta''[t] r'[t] + \vartheta''[t] r[t]\right\} \quad \text{/.} \\
& \quad \{r''[t] \rightarrow 0, \ r'[t] \rightarrow 0\} \quad \text{/.} \quad \{\vartheta'[t] \rightarrow v / R, \ r[t] \rightarrow R\}
\end{align*}
\]

\[
\text{Out}[7] := \left\{ \frac{v^2}{R}, \ R \vartheta''[t] \right\}
\]

Acting vertically and horizontally, respectively, at the bottom of the arc.

Use conservation of energy equations to get \(\vartheta''[t]\). Let \(\theta\) be zero pointing from the center of the arc to the ground and the gravity potential measured from the bottom of the arc.
\textbf{Part B}

\begin{verbatim}
In[12]:= Vi = m g H; 
   Ti = 0; 
   Vf = m g h; 
   Tf = \frac{1}{2} m v^2; 
   eq = Vi + Ti == Vf + Tf 

Out[12]= G H m == g h m + \frac{1}{2} m v^2 

This is the magnitude of the velocity.

Out[12]= v \to \sqrt{2 \sqrt{-g h + g H}}

It leaves at 50° from the horizontal, so the component form is:

Out[12]= vvec1 = \{\cos[50°], \sin[50°]\} v /. vvec1

\end{verbatim}
Part C

The acceleration of the marble as it peaks in free flight is just gravity.

Part D

Find the time for the marble to hit the ground.

\[
\text{time} = \text{Solve}\left[0 = h + \text{vvec1[[2]]} \, t - \frac{1}{2} \, g \, t^2, t\right] / / \text{Last} / / \text{Last}
\]

\[
t \to \frac{2 \sqrt{-g (h - H) \cos[40°] + \sqrt{8 \, g \, h - 8 \, g \, (h - H) \cos[40°]}}}{2 \, g}
\]

Plug this into the distance kinematics.

\[
x_{\text{dist1}} = \text{vvec1[[1]]} \, t / . \text{time} / / \text{Simplify}
\]

\[
\frac{1}{g} \left[2 \sqrt{-g (h - H) \sin[40°] \left(\sqrt{g \, (h + H) \cos[40°]} + \sqrt{g \, (H \cos[40°]^2 + h \sin[40°]^2)}\right)}\right]
\]

Problem 2

Part A

Conserve energy, then use the single rotated frame formulas. I will assume that \(r\) is very small relative to \(R\).

\[
\text{Vi} = m \, g \, H;
\]

\[
\text{Ti} = 0;
\]

\[
\text{Vf} = 0;
\]

\[
\text{Tf} = \frac{1}{2} \, m \, v^2 + \frac{1}{2} \, \text{II} \, \omega^2 / . \, \omega \to \frac{v}{r};
\]

\[
\text{eq} = \text{Vi} + \text{Ti} = \text{Vf} + \text{Tf}
\]

\[
\text{g \, H \, m} = \frac{m \, v^2}{2} + \frac{\text{II} \, v^2}{2 \, r^2}
\]

\[
\text{solv2} = (\text{Solve}[\text{eq}, v] / / \text{Last}) / . \text{II} \to 2 / 5 \, m \, \text{r}^2
\]

\[
\{v \to \sqrt{\frac{10}{7} \, \sqrt{g \, \sqrt{H}}}\}
\]

The acceleration of the marble at the bottom of the arc is
In[27]:= \[ acc2 = \{ -x''[t] + \theta'[t]^2 x[t], 2 \theta' [t] r'[t] + \theta''[t] r[t] \} /.
\{ r''[t] \to 0, r'[t] \to 0 \} / (\theta'[t] \to v, (R-r), r[t] \to (R-r))\]

Out[27]= \[ \{ \begin{align*} v^2 \\
-\frac{v}{r + R} \end{align*} \right\} \]

Acting vertically and horizontally, respectively, at the bottom of the arc.

Use conservation of energy equations to get \( \theta''[t] \). Let \( \theta \) be zero pointing from the center of the arc to the ground and the gravity potential measured from the bottom of the arc.

In[28]:= \[ eq2 = \frac{1}{2} m x'[t]^2 + m g ((R-r) - (R-r) \cos[\theta[t]]) \to 0 / . \ x'[t] \to \theta'[t] (R-r) \]

Out[28]= \[ g m (-x + R - (-x + R) \cos[\theta[t]]) + \frac{1}{2} m (-r + R)^2 \theta'[t]^2 = 0 \]

The total energy is unchanging in time as well

In[29]:= \[ sol\theta''d2 = Solve[D[eq2, t], \theta''[t]] / Last // Last \]

Out[29]= \[ \theta''[t] \to \frac{g \sin[\theta[t]]}{r - R} \]

So the acceleration becomes

In[30]:= \[ acc2 = acc2 / . solv2 / . sol\theta''d2 / \theta[t] \to 0 \]

Out[30]= \[ \{ \frac{10 g H}{7 (-r + R)}, 0 \} \]

Its magnitude is

In[31]:= \[ magAcc2 = \sqrt{(acc2[[1]])^2 + acc2[[2]]^2} \]

Out[31]= \[ \frac{10}{7} \sqrt{\frac{g^2 H^2}{(-r + R)^2}} \]

Part B

In[32]= \[ Vi = m g H; \]
\[ Ti = 0; \]
\[ Vf = m g h; \]
\[ Tf = \frac{1}{2} m v^2 + \frac{1}{2} I I \omega^2 / . \omega \to \frac{v}{r}; \]
\[ eq = Vi + Ti = Tf + Vf \]

Out[36]= \[ g H m = g h m + \frac{m v^2}{2} + \frac{I I v^2}{2 r^2} \]

This is the magnitude of the velocity.
In[37]:= \text{solv2} = \text{Solve[eq, v] // Last} /. \text{II} \rightarrow 2 / 5 \text{m r}^2

It leaves at 50° from the horizontal, so the component form is:

In[38]:= \text{vvec2} = (\text{Cos}[50°], \text{Sin}[50°]) \cdot \text{solv}

Part C

The acceleration of the marble as it peaks in free flight is just gravity.

Part D

Find the time for the marble to hit the ground.

In[39]:= \text{time} = \text{Solve[0 = h + vvec2[[2]] t - \frac{1}{2} g t^2, t] // Last} // Last

Out[39]= t \rightarrow -\frac{1}{14 g \sqrt{m}} \left[-2 \sqrt{70} \sqrt{-g \ (h - H) m \ \text{Cos}[40°]} + \sqrt{392 g h m - 280 g \ (h - H) m \ \text{Cos}[40°]^2} \right]

Plug this into the distance kinematics.

In[40]:= \text{xdist2} = vvec2[[1]] \cdot \text{time} // \text{Simplify}

Out[40]= \frac{1}{7 g m} 2 \sqrt{5} \sqrt{-g \ (h - H) m}

\left[\sqrt{5} \sqrt{-g \ (h - H) m \ \text{Cos}[40°]} - \sqrt{g m \ (5 H \ \text{Cos}[40°]^2 + h \ (7 - 5 \ \text{Cos}[40°]^2)} \right] \ \text{Sin}[40°]

Compare the point marble to sphere marble

In[41]:= \text{xdist1} > \text{xdist2} /. \{\text{H} \rightarrow 1, \text{h} \rightarrow 1/2, \text{g} \rightarrow 9.81, \text{m} \rightarrow 1\}

Out[41]= \text{True}

In[42]:= \text{magAcc1} > \text{magAcc2} /. \{\text{H} \rightarrow 1, \text{h} \rightarrow 1/2, \text{g} \rightarrow 9.81, \text{m} \rightarrow 1, \text{R} \rightarrow 1, \text{r} \rightarrow .1\}

Out[42]= \text{True}

The momentum causing the ball to rotate robs momentum from the translational motion.
Problem 3

See hand written work.

Problem 4

\[
\text{Power} = F \times v
\]

\[
\text{Power in hp} = \frac{\text{force in lbf} \times \text{vel in mph}}{375}
\]

\[
F_d = \frac{1}{2} \rho \nu^2 c_d A
\]

In[43]:= \text{cons} = \{\text{weight}_{\text{truck}} \rightarrow 2000 \times 50, \ \theta \rightarrow 15^\circ, \ c \rightarrow 0.8, \ v \rightarrow 25\}

Out[43]= \{\text{weight}_{\text{truck}} \rightarrow 100\,000, \ \theta \rightarrow 15^\circ, \ c \rightarrow 0.8, \ v \rightarrow 25\}

In[44]:= \text{airdrag} = \frac{1}{2} \left(0.0023769 \ \text{slugs per ft}^3\right) \left(v \times 1.4667 \ \text{ft per sec}\right)^2 c \left(9 \times 9 \text{ft}^2\right) / \text{cons}

\text{airdrag} = \% /\text{ First;}

103.542 \text{ ft slugs per sec}^2

Out[44]=

In[46]:= \text{Power} = \left(\text{weight}_{\text{truck}} \sin[\theta] + \text{airdrag}\right) v / \text{cons}

\text{Power} = 1732.36

Out[46]=

With no wind, the power required is:

In[47]:= \text{Power} = \frac{\left(\text{weight}_{\text{truck}} \sin[\theta] + 0\right) v}{375} / \text{cons} /\text{N}

Out[47]=

\text{Power} = 1725.46