ME2302: Quiz Project # 6

The two bars $A$ and $B$ shown make up the main parts of the Quick Return Mechanism. Pin $p$ is fixed to $A$ and bar $B$ has a slot that allows pin $p$ to move freely along the slot. Assume $\dot{\theta} = \omega_A \frac{\pi}{180}$, a constant. Measure the angle $\theta$ from $\hat{n}_1$. Let the angular speed of bar $B$ be given by $\dot{\beta} = \omega_B$, not constant. Measure the angle $\beta$ from $\hat{n}_1$ as well. The work here is all kinematical and is essentially part of step 5 of the six step process. Use Mathematica and the vector tools. Follow previous quizzes and such posted on the class website.

1. Determine the following quantities as a function of $\theta$:
   (a) the magnitude of the absolute velocity of pin $p$,
   (b) the magnitude of the absolute acceleration of pin $p$,
   (c) the velocity of pin $p$ as seen from bar $B$ expressed in vectors in $B$,
   (d) the acceleration of pin $p$ as seen from bar $B$ expressed in vectors in $B$,
   (e) the plots all of these quantities,
   (f) the point on each bar $A$ and $B$ that experiences the most acceleration during the motion, and
   (g) explain via analysis why it is called a "Quick Return" mechanism. Hint: look at $\beta$ and its derivatives.

2. Animate the device allowing the parameters $R$ and $D$ to vary.

3. Find the ratio of $R/D$ that allows the quickest return.

Load the vector tools

This will be a relative path on your machine. Use the get file path menu item under insert or use the notebook relative path I sent in an email. I typed this text in then selected the cell bar on the right and hit [command] 7 on Mac or [Alt] 7 on PC to make into a text cell. In order to execute any cell you must put the cursor in the cell and hit [shift] [return] or on the numeric keypad [enter].

In[1]:= << "/Users/alanbarhorst/Book/Rev2010/dynmath/math3/vectors3"

These Engineering Vector algorithms are copyright Alan A. Barhorst
Problem 1

Define frames

For the frames, \( a[1] \) along crank (o to p), \( a[2] \) to the right, \( b[1] \) from b to p, \( b[2] \) down to the left; at o, \( n[1] \) to the right and \( n[2] \) up.

\[
\begin{align*}
\text{In[2]} &= a[x_] := \text{unitVector}[A, a, x] \\
& b[x_] := \text{unitVector}[B, b, x] \\
& n[x_] := \text{unitVector}[N, n, x]
\end{align*}
\]

Define transformations

\[
\begin{align*}
\text{In[5]} &= \text{trans} = \\
& \{a[1] \rightarrow \cos[\theta[t]] \ n[1] + \sin[\theta[t]] \ n[2], a[2] \rightarrow -\sin[\theta[t]] \ n[1] + \cos[\theta[t]] \ n[2], a[3] \rightarrow n[3], b[1] \rightarrow \cos[\beta[t]] \ n[1] + \sin[\beta[t]] \ n[2], b[2] \rightarrow -\sin[\beta[t]] \ n[1] + \cos[\beta[t]] \ n[2], b[3] \rightarrow n[3]\}
\end{align*}
\]

Angular velocity vectors

\[
\begin{align*}
\text{In[6]} &= \text{NwA} = \omega[N, A] = \theta'[t] \ a[3] \\
\text{Out[6]} &= \hat{a}_3 \theta'[t] \\
\text{Out[7]} &= \hat{a}_3 \theta'[t] \\
\text{In[7]} &= \theta[t_] = \omega A t \\
\text{Out[7]} &= t \ \omega A \\
\text{In[8]} &= \text{SetAttributes[\{\omega A, R, Dd\}, \text{Constant}]}
\end{align*}
\]

\[
\begin{align*}
\text{In[8]} &= \text{NwB} = \omega[N, B] = \beta'[t] \ b[3] \\
\text{Out[9]} &= \hat{b}_3 \beta'[t] \\
\text{Out[15]} &= \hat{b}_3 \beta'[t]
\end{align*}
\]

Part a)

\[
\begin{align*}
\text{In[10]} &= \text{OrP} = R a[1] \\
\text{Out[10]} &= R \hat{a}_1 \\
\text{Out[16]} &= R \hat{a}_1
\end{align*}
\]

The velocity of p is given by
In[11]:= \( \mathbf{OvP} = \mathbf{DvDt}[\mathbf{N}, \mathbf{OrP}] \)

Out[11]= \( R \ \omega \ \mathbf{A} \ \mathbf{\hat{a}}_2 \)

Out[19]= \( R \ \omega \ \mathbf{A} \ \mathbf{\hat{a}}_2 \)

Its magnitude is

In[12]:= \( \text{magVp}[\mathbf{R}_-, \omega \mathbf{A}_-] = \sqrt{\mathbf{OvP}.\mathbf{OvP}} \)

Out[12]= \( \sqrt{R^2 \ \omega^2} \)

**Part b)**

The Acceleration of \( \rho \) is given by

In[13]:= \( \mathbf{OxP} = \mathbf{DvDt}[\mathbf{N}, \mathbf{OvP}] \)

Out[13]= \( -R \ \omega \ \mathbf{A}^2 \ \mathbf{\hat{a}}_1 \)

Its magnitude is

In[14]:= \( \text{magXp}[\mathbf{R}_-, \omega \mathbf{A}_-] = \sqrt{\mathbf{OxP}.\mathbf{OxP}} \)

Out[14]= \( \sqrt{R^2 \ \omega^4} \)

**Part c)**

To answer part c we need the expressions for \( l[t] \) and \( l'[t] \)

Write the vector loop

In[15]:= \( \mathbf{vloop} = \mathbf{Dd} \ \mathbf{n}[1] + l[t] \ \mathbf{b}[1] - R \ \mathbf{a}[1] \)

Out[15]= \( -R \ \mathbf{\hat{a}}_1 + l[t] \ \mathbf{\hat{b}}_1 + \mathbf{Dd} \ \mathbf{\hat{n}}_1 \)

Differentiate once, angular velocities defined above.

In[16]:= \( \mathbf{vloopD} = \mathbf{DvDt}[\mathbf{N}, \mathbf{vloop}] \)

Out[16]= \( -R \ \omega \ \mathbf{A} \ \mathbf{\hat{a}}_2 + \mathbf{\hat{b}}_1 \ l'[t] + l[t] \ \mathbf{\hat{b}}_2 \ \beta'[t] \)

Out[30]= \( -R \ \omega \ \mathbf{A} \ \mathbf{\hat{a}}_2 + \mathbf{\hat{b}}_1 \ l'[t] + l[t] \ \mathbf{\hat{b}}_2 \ \beta'[t] \)

Differentiate again

In[17]:= \( \mathbf{vloopDD} = \mathbf{DvDt}[\mathbf{N}, \mathbf{vloopD}] // \text{Expand} \)

Out[17]= \( R \ \omega \ \mathbf{A}^2 \ \mathbf{\hat{a}}_1 + 2 \ \mathbf{\hat{b}}_2 \ l'[t] \ \beta'[t] - l[t] \ \mathbf{\hat{b}}_1 \ \beta'[t]^2 + \mathbf{\hat{b}}_1 \ l''[t] + l[t] \ \mathbf{\hat{b}}_2 \ \beta''[t] \)

Find position information as follows, we are using the 2nd and third quadrant solution for \( \beta[t] \).

In[18]:= \( l\beta\text{solve} = \)

\( \{\text{Solve}[\{\mathbf{vloop} . \mathbf{b}[1] = 0, \mathbf{vloop} . \mathbf{b}[2] = 0\} /.\ \text{trans}, \{l[t], \ \beta[t]\}] \}/.\ C[1] \rightarrow 0\}[[1]] \)

Find velocity information as follows
In[19]:= \textbf{lDdsolv} = (\texttt{Solve[\{vloopD.b[1] == 0, vloopD.b[2] == 0}, \{l'[t], \beta'[t]\}]) \quad // \textbf{First}

Out[19]= \textbf{\{l'[t] \rightarrow R \omega A \hat{a}_2 \cdot \hat{b}_2, \beta'[t] \rightarrow \frac{R \omega A \hat{a}_2 \cdot \hat{b}_2}{l[t]} \}}

The required answer is \textbf{l'[t]} \textbf{b[1]}

**Part d)**

Find acceleration information as follows

In[20]:= \textbf{lDDdsolv} = (\texttt{Solve[\{vloopDD.b[1] == 0, vloopDD.b[2] == 0}, \{l''[t], \beta''[t]\}]) \quad // \textbf{First}

Out[20]= \textbf{\{l''[t] \rightarrow -R \omega A^2 \hat{a}_1 \cdot \dot{\hat{b}}_1 + l[t] \beta'[t]^2, \beta''[t] \rightarrow \frac{-R \omega A^2 \hat{a}_1 \cdot \dot{\hat{b}}_2 - 2 l'[t] \beta'[t]}{l[t]} \}}

The required answer is \textbf{l''[t]} \textbf{b[1]}

**Part e)**

In[21]:= \textbf{Manipulate[Plot[magVp[R, \omega A], \{t, 0, 2 \pi / \omega A\},}

\texttt{Frame \rightarrow True, FrameLabel \rightarrow \{"t (s)", \"|\dot{v}| (m/s)\"},

\texttt{PlotLabel \rightarrow "Magnitude of the velocity of p"], ((R, 1), \{.1, 10\}, \{\{\omega A, 1\}, 1, 10\})]}

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{Magnitude_of_the_velocity_of_p.png}
\caption{Magnitude of the velocity of p}
\end{figure}

Out[21]=
In[22]:= Manipulate[Plot[magXp[R, ωA], {t, 0, 2 Pi/ωA},
   Frame -> True, FrameLabel -> {"t (s)", "|\ddot{\mathbf{a}}| (m/s^2)"},
   PlotLabel -> "Magnitude of the acceleration of \(\mathbf{p}\)",
   {{R, 1}, .1, 10}, {{ωA, 1}, 1, 10}]

Out[22]=

In[23]:= lplot[Dd_, R_, ωA_, t_] = l[t] /. lβsolv
In[24]:= Manipulate[Plot[lplot[Dd, R, ωA, t], {t, 0, 2 Pi / ωA}, Frame -> True, FrameLabel -> "t (s)", "l[t] (m)"], PlotLabel -> "Slot Position of p"], {{Dd, 1}, .1, 10}, {{R, .5}, .1, 10}, {{ωA, 1}, 1, 10}]

Out[24]=

In[25]:= lDplot[Dd_, R_, ωA_, t_] = l'[t] /. lβDsolve /. trans /. lβsolve
In[26]:  Manipulate[Plot[Ddplot[Dd, R, ωA, t], {t, 0, 2 Pi/ωA}, Frame -> True, 
FrameLabel -> {"t (s)", "l'[t] (m/s)"}, PlotLabel -> "Slot speed of p"], 
{(Dd, 1), .1, 10}, {(R, .5), .1, 10}, {(ωA, 1), 1, 10}]

Out[26]=

In[27]:  lDDplot[Dd__, R__, ωA__, t__] = l''[t] /. lβDDsolv /. lβDsolv /. trans /. lβsolv
Manipulate[Plot[lDDplot[Dd, R, wA, t], {t, 0, 2 Pi / wA}, Frame -> True, FrameLabel -> {"t (s)", "1' t (m/s^2)"}, PlotLabel -> "Slot acceleration of p"], {{Dd, 1}, .1, 10}, {{R, .5}, .1, 10}, {{wA, 1}, 1, 10}]

Part f)

Bar A is moving a constant angular velocity so the tip of the bar will experience the most $\omega^2 R$ type acceleration because in this case $R$ would be the entire length of bar A.

Bar B experiences angular acceleration. Therefore we examine

SetAttributes[{X}, Constant]

$B_x P_b = DvD_t[N, DvD_t[N, X b[1]]] \text{ // Expand}

$\beta_1'[t]^2 + X \beta_2''[t]

where $X$ is just any location on bar B. The magnitude of the acceleration is

$magXPb[X, Dd, R, wA, t] = Sqrt[BxPb.BxPb] \text{ // l\textbetaDDsolv // l\textbetaDsolve // trans // l\textbetaDsolve // Simplify}

$\frac{1}{2 \sqrt{2}} \left( R^2 X^2 \omega A^4 \left( 4 Dd^6 - 5 Dd^4 R^2 + 28 Dd^2 R^4 + 8 R^6 - 8 Dd R^3 (3 Dd^2 + 4 R^2) \cos[t \omega A] - 4 Dd^4 (Dd^4 - 3 Dd^2 R^2 - 5 R^4) \cos[2 t \omega A] - 8 Dd^3 R^3 \cos[3 t \omega A] + Dd^4 R^2 \cos[4 t \omega A] \right) \right) \left( Dd^2 + R^2 - 2 Dd R \cos[t \omega A] \right)^4$

Which will be max when $X = L$ of bar B.
Part g)

If we look at the expression for $\beta'[t]$

\[
\text{In}[32]= \text{Manipulate} \left[ \text{Plot} \left[ \text{magXPb} [X, Dd, R, \omega A, t], \{t, 0, 2 \pi / \omega A \} \right], \text{Frame} \to \text{True}, \text{FrameLabel} \to \{\text{"t (s)"}, \text{"l' (m/s^2)"}\}, \text{PlotLabel} \to \text{"Acceleration magnitude of point X along bar B"}, \{\{X, .1\}, 0, 10\}, \{\{Dd, 1\}, .1, 10\}, \{\{R, .5\}, .1, 10\}, \{\{\omega A, 1\}, 1, 10\} \right]
\]

We see that it is larger when $l[t]$ is smaller so the bar $B$ flies back much faster than it advances. The closer point $p$ is to the location of the anhcer of bar $B$ the faster the machine will flyback.

Problem 2

In this solution graphics primitives are manipulated instead of lines.

Bar A coordinates

\[
\text{In}[34]= \text{XA1} [Ra_, La_] = -Ra; \quad \text{YA1} [Ra_, La_] = -Ra;
\]
In[36]:= \[XA2[\text{Ra}, \text{La}] = \text{La} + \text{Ra};
YA2[\text{Ra}, \text{La}] = \text{Ra};
\]
Bar B coordinates and its angle

In[38]:= \[XB1[\text{Rb}, \text{Lb}, \text{Dd}] = \text{Dd} + \text{Rb};
XB2[\text{Rb}, \text{Lb}, \text{Dd}] = -\text{Rb};
\]
In[40]:= \[\beta b[\text{Dd}, \theta] = \beta[t] / . \text{betaSol} / . \{t \to \theta\}
\]

Out[42]= \[\text{ArcTan}\left[\frac{\text{Dd} - \text{R Cos}[\theta]}{\sqrt{\text{Dd}^2 - 2 \text{Dd} \text{R Cos}[\theta] + \text{R}^2 \text{Cos}[\theta]^2 + \text{R}^2 \text{Sin}[\theta]^2}}\right]
\]

In[43]= Manipulate[
Show[
{EdgeForm[Thick], Red,
Rotate[
  Rectangle[\{\text{Xa1[\text{Ra}, \text{La}], \text{Ya1[\text{Ra}, \text{La}]},
    \{\text{Xa2[\text{Ra}, \text{La}], \text{Ya2[\text{Ra}, \text{La}]}, \text{RoundingRadius} \to \text{Ra}, \theta, \{0, 0\}}
  ],
Black,
Rotate[Arrow[\{\{0, 0\}, \{6 \text{Ra}, 0\}\}], \theta, \{0, 0\}],
Rotate[Arrow[\{\{0, 0\}, \{0, 6 \text{Ra}\}\}], \theta, \{0, 0\}],
{PointSize[.02], Black, Point[\{0, 0\}]}]
},
Graphics[
{EdgeForm[Thick], Green,
Rotate[Rectangle[\{\text{XB1[\text{Rb}, \text{Lb}, \text{Dd}], \text{YB1[\text{Rb}, \text{Lb}, \text{Dd}]},
    \{\text{XB2[\text{Rb}, \text{Lb}, \text{Dd}]}, \text{YB2[\text{Rb}, \text{Lb}, \text{Dd}]}, \text{RoundingRadius} \to \text{Rb}, \pi + \beta b[\text{Dd}, \theta, \text{Dd}, 0\}]
  ],
Black,
Rotate[Arrow[\{\{\text{Dd}, 0\}, \{\text{Dd} - 6 \text{Rb}, 0\}\}], \pi + \beta b[\text{Dd}, \theta, \{\text{Dd}, 0\}],
Rotate[Arrow[\{\{\text{Dd}, 0\}, \{\text{Dd} - 6 \text{Rb}\}\}], \pi + \beta b[\text{Dd}, \theta, \{\text{Dd}, 0\}],
{PointSize[.02], Black, Point[\{\text{Dd}, 0\}]},
{PointSize[.03], Black, Point[\{\text{R Cos}[\theta], \text{R Sin}[\theta]\}]]
}]
},
PlotRange \to \{\{-\text{La} - 3 \text{Ra}, \text{Dd} + 3 \text{Rb}\}, \{-\text{La} - 5 \text{Ra}, \text{La} + 5 \text{Ra}\}\}
], \{(\text{Dd}, 2), \{5, 10\}, \{(\text{R}, 1), \{.1, 10\}, \{(\text{Ra}, .1), 0, 1\},
\{(\text{La}, 1.5), 1, 10\}, \{(\text{Rb}, .1), 0, 1\}, \{(\text{Lb}, 4), 1, 10\}, \{(\theta, 0, 4 \pi)\]
**Problem 3**

The maximum $\beta'[t]$ occurs when $l[t]$ is smalles or when $\theta = 0, 2\pi$, etc.

\[
\beta' = \left(\frac{R \omega A}{-Dd + R}\right) \quad \text{Simplify} \quad \theta \rightarrow 0 \quad \text{Simplify}
\]

So if this denominator goes to zero than the return will be the quickest. So the ratio R/D must approach 1 as much as reasonably possible for the device at hand.