ME2302: Test # 2

1. (10 pts)

(a) A standard system of coordinates that is parameterized by a radius vector and elevation angle and an azimuthal or heading angle is called by what name?
   \[ \textit{Spherical coordinates} \]

(b) For a system moving on a curvilinear path, what standard set of coordinates is a natural fit for such motion?
   \[ \textit{Tangent and normal, or path coordinates} \]

(c) Cylindrical coordinates describe the position of a point with what three parameters?
   \[ r, \theta, z \quad \text{or} \quad r, \theta, h \]

(d) If the elevation angle is not present (equal to zero) in a cylindrical coordinate system what is the resulting planar system called?
   \[ \textit{Polar coordinates} \]

(e) Adding what coordinate to a polar coordinate system provides a cylindrical coordinate system?
   \[ z \quad \text{or} \quad h \]
2. (15 pts) The system below for the instant shown has the block moving to the right at \( v = R(1) \) m/s. The lengths are given by \( R = R(2)R(3)/10 \) m, \( L = 2R \). For \( \theta = R(4)R(5)^\circ \), find the angular velocity (indicate CW or CCW) of each of the bars at this instant in time. Where \( R(i) \) is the \( i \)th nonzero digit of your R number. Thus, if your R-number is \( R = 0234607 \), then \( R(1) = 2 \) and \( R(2)R(3) = 34 \), etc.
3. (15 pts) An industrial robot is in the configuration shown for the instant described. Find the magnitude of the velocity of tip \( p \). At this instant \( \theta = 30^\circ \) and increasing at a rate of \( R(1) \text{ rad/s} \), \( \beta = 110^\circ \), \( D = R(3) \text{ m/s} \), \( R = 1 \text{ m} \), and \( L = 2 \text{ m} \). Where \( R(i) \) is the \( i \)th nonzero digit of your \( R \) number. Thus, if your \( R \)-number is \( R = 0234607 \), then \( R(1) = 2 \) and \( R(2)R(3) = 34 \), etc.

\[
\bar{\nu}_p = \frac{d}{dt} \bar{r}_p + a \bar{r}_p = 2R \hat{a}_1 + L \hat{b}_1
\]

\[
|\bar{\nu}_p| = \left[ (2R \dot{\theta})^2 + 2RL \dot{\theta} \dot{\beta} \cos(110^\circ) + (L \dot{\beta})^2 \right]^{1/2}
\]
4. (60 pts) A proposed paint shaking system is shown below. The paint can height $H$ would be attached at $p$ midway on the top bar as shown. Links $a-b$ and $c-d$ are equal length $L$. Link $c-b$ is a compressible/extensible hydraulic cylinder element of nominal length $D$, extension $x(t)$ that shakes the system by extending $x(t)$ in a known/prescribed cyclic fashion. Find:

(a) the transformation expressions relating frames $A$, $B$, $C$ and $O$, local vectors to the $N$ vectors, use positive right-hand sense for the frames and angles as drawn,

(b) the number of degrees of freedom and the vector loop expression(s) that exist for this device in terms of local vectors and base $N$ frame vectors, but contain no explicit trigonometric functions; in other words write them in terms of local vectors as needed,

(c) expressions relating $\dot{\theta}$, $\gamma$, $\beta$, and $\phi$ to $\dot{x}$ and $x$, in terms of unresolved vector dot products,

(d) expressions relating $\ddot{\theta}$, $\ddot{\gamma}$, $\ddot{\beta}$, and $\ddot{\phi}$ to $\ddot{x}$, $\dot{x}$ and $x$, in terms of unresolved vector dot products,

(e) the expression for the absolute velocity of point $p$ in the center of the paint can, but using local frame vectors, symbols representing items found above need not be replaced and can be left as symbols, and

(f) the expression for the absolute acceleration of point $p$ but using local frame vectors, symbols representing items found above need not be replaced and can be left as symbols.

This would be mostly be step 5 of the 6 step process.

\[
\begin{align*}
\hat{\gamma}_1 &= \cos \gamma \hat{n}_1 + \sin \gamma \hat{n}_2 \\
\hat{\gamma}_2 &= -\sin \gamma \hat{n}_1 + \cos \gamma \hat{n}_2 \\
\hat{\gamma}_3 &= \hat{n}_3 \\
\hat{\gamma}_4 &= \cos \theta \hat{n}_1 + \sin \theta \hat{n}_2, \quad \hat{\gamma}_5 = -\sin \theta \hat{n}_1 + \cos \theta \hat{n}_2, \quad \hat{\gamma}_6 = \hat{n}_3 \\
\end{align*}
\]

\[
\begin{align*}
\dot{\gamma}_1 &= \frac{d}{dt} \left[ \cos \gamma \hat{n}_1 + \sin \gamma \hat{n}_2 \right] \\
\dot{\gamma}_2 &= \frac{d}{dt} \left[ -\sin \gamma \hat{n}_1 + \cos \gamma \hat{n}_2 \right] \\
\dot{\gamma}_3 &= \frac{d}{dt} \hat{n}_3 \\
\dot{\gamma}_4 &= \frac{d}{dt} \left[ \cos \theta \hat{n}_1 + \sin \theta \hat{n}_2 \right], \quad \dot{\gamma}_5 = -\sin \theta \hat{n}_1 + \cos \theta \hat{n}_2, \quad \dot{\gamma}_6 = \hat{n}_3 \\
\end{align*}
\]

\[\begin{align*}
\otimes (p + x) \hat{\gamma}_5 - C \hat{\gamma}_5 - A \hat{\gamma}_4 &= 0 \\
\hat{\gamma}_5 &= \hat{\gamma}_3 \end{align*}\]
problem \(4\)

\( \nabla \phi = 0 \rightarrow C \phi = 0 \)

\( \mathbf{v}_{\text{loop1}} = \mathbf{v}_{\text{1}} + (D+x)\mathbf{v}_{\text{2}} - C\mathbf{v}_{\text{2}} = 0 \)

\( \text{dot with } \mathbf{v}_{\text{1}} \Rightarrow \dot{\phi} = \frac{-x(\mathbf{v}_{\text{1}} \cdot \mathbf{v}_{\text{2}})}{(D+x)(\mathbf{v}_{\text{1}} \cdot \mathbf{v}_{\text{2}})} \) \( \text{(5)} \)

\( \text{dot with } \mathbf{v}_{\text{2}} \Rightarrow \dot{\phi} = -\frac{x}{C(\mathbf{v}_{\text{1}} \cdot \mathbf{v}_{\text{2}})} \) \( \text{(5)} \)

\( \mathbf{v}_{\text{loop2}} = L \mathbf{v}_{\text{2}} + E \mathbf{v}_{\text{2}} + L \mathbf{v}_{\text{2}} = 0 \)

\( \mathbf{v}_{\text{1}} \mathbf{v}_{\text{2}} - E \mathbf{v}_{\text{2}} + L \mathbf{v}_{\text{2}} = 0 \)

\( \text{dot with } \mathbf{v}_{\text{2}} \Rightarrow \beta = \frac{L \mathbf{v}_{\text{2}}}{E(\mathbf{v}_{\text{1}} \cdot \mathbf{v}_{\text{2}})} \) \( \text{(5)} \)

\( \text{dot with } \mathbf{v}_{\text{2}} \Rightarrow \dot{\phi} = -\frac{x}{C(\mathbf{v}_{\text{1}} \cdot \mathbf{v}_{\text{2}})} \) \( \text{(5)} \)

\( \mathbf{v}_{\text{loop1}} = \mathbf{v}_{\text{1}} + (D+x)\mathbf{v}_{\text{2}} - (D+x)\mathbf{v}_{\text{2}} - C\mathbf{v}_{\text{2}} = 0 \)

\( \text{dot with } \mathbf{v}_{\text{1}} \Rightarrow \dot{\phi} = \frac{-x(\mathbf{v}_{\text{1}} \cdot \mathbf{v}_{\text{2}})(x(\mathbf{v}_{\text{1}} \cdot \mathbf{v}_{\text{2}}) + (D+x)\mathbf{v}_{\text{2}})}{(D+x)(\mathbf{v}_{\text{1}} \cdot \mathbf{v}_{\text{2}})} \) \( \text{(5)} \)

\( \text{dot with } \mathbf{v}_{\text{2}} \Rightarrow \dot{\phi} = \frac{x + (D+x)x^2 + Cx^2(\mathbf{v}_{\text{1}} \cdot \mathbf{v}_{\text{2}})}{C(\mathbf{v}_{\text{1}} \cdot \mathbf{v}_{\text{2}})} \)

\( \mathbf{v}_{\text{loop2}} \Rightarrow L \mathbf{v}_{\text{2}} - L \mathbf{v}_{\text{2}} - E \mathbf{v}_{\text{2}} + L \mathbf{v}_{\text{2}} = 0 \)

\( \text{dot with } \mathbf{v}_{\text{2}} \Rightarrow \beta = \frac{L \mathbf{v}_{\text{2}}}{E(\mathbf{v}_{\text{1}} \cdot \mathbf{v}_{\text{2}})} \) \( \text{(5)} \)

\( \text{dot with } \mathbf{v}_{\text{2}} \Rightarrow \dot{\phi} = -\frac{x}{C(\mathbf{v}_{\text{1}} \cdot \mathbf{v}_{\text{2}})} \) \( \text{(5)} \)

\( \mathcal{O} = \mathbf{A} \mathbf{v} + 1 \mathbf{v} + \mathbf{v} \) \( \text{known above} \) \( \text{+10} \)

\( \mathcal{O} = L \mathbf{v} \mathbf{v} - \mathbf{v} \mathbf{v} - \mathbf{v} \) \( \text{known above} \) \( \text{+10} \)

\( \mathbf{v} = L \mathbf{v} \mathbf{v} - \mathbf{v} \mathbf{v} - \mathbf{v} \) \( \text{+10} \)

\( \mathbf{v} = L \mathbf{v} \mathbf{v} - \mathbf{v} \mathbf{v} - \mathbf{v} \) \( \text{+10} \)

\( \mathbf{v} = L \mathbf{v} \mathbf{v} - \mathbf{v} \mathbf{v} - \mathbf{v} \) \( \text{+10} \)
5. (Bonus 10pts) Set up the transcendental equations that can be used to find the angles $\theta$, $\gamma$, $\beta$, and $\phi$ given $x$ is known. Do not try to solve them.

\[ \sqrt{\text{loop 1}} = (0 + x) \hat{\mathbf{g}}_1 - C \hat{\mathbf{g}}_1 - A \hat{\mathbf{a}}_1 = 0 \]

\[ \text{dot with } \hat{\mathbf{g}}_1 : (0 + x)(\hat{\mathbf{g}}_1 \cdot \hat{\mathbf{g}}_1) - C(\hat{\mathbf{g}}_1 \cdot \hat{\mathbf{g}}_1) - A = 0 \]

\[ \text{or} \]

\[ \text{dot with } \hat{\mathbf{g}}_2 : (0 + x)(\hat{\mathbf{g}}_2 \cdot \hat{\mathbf{g}}_1) - C(\hat{\mathbf{g}}_1 \cdot \hat{\mathbf{g}}_2) = 0 \]

\[ (0 + x)(\sin\theta) - C(\sin\phi) = 0 \]

\[ \text{or} \]

\[ \sqrt{\text{loop 2}} = L \hat{\mathbf{a}}_1 + E \hat{\mathbf{a}}_2 + (\hat{\mathbf{g}}_1 - \beta \hat{\mathbf{a}}_1) = 0 \]

\[ \text{dot with } \hat{\mathbf{a}}_1 : L(\hat{\mathbf{a}}_1 \cdot \hat{\mathbf{a}}_1) + E(\hat{\mathbf{a}}_2 \cdot \hat{\mathbf{a}}_1) + L(\hat{\mathbf{g}}_1 \cdot \hat{\mathbf{a}}_1) - \beta = 0 \]

\[ L \cos\gamma + E \cos\beta + L \cos\phi - \beta = 0 \]

\[ \text{or} \]

\[ \text{dot with } \hat{\mathbf{g}}_2 : L(\hat{\mathbf{g}}_1 \cdot \hat{\mathbf{g}}_2) + E(\hat{\mathbf{g}}_2 \cdot \hat{\mathbf{g}}_2) + L(\hat{\mathbf{g}}_1 \cdot \hat{\mathbf{g}}_2) = 0 \]

\[ L(\sin\phi) + E(-\cos\beta) + C(-\sin\phi) = 0 \]