1. (10 pts) In the textbook what is a) the name of the hypothetical consulting company, b) what was the contents of the packages being transferred on the roller table example, c) Sammy has issues transporting what items on what vehicle, d) what action of a pet caused Sue to frown, and e) what stunt does Billy perform?

a) ACME
b) Bread - Truly Taste Bread Company
c) Blocks stacked one on the other on a Flatbed Truck
d) Dog moves while in a untied boat
e) Crashes his car into another car
2. (15 pts) Block $m_1$ is initially moving to the right at speed $v = R(1)R(3)$ m/s. Block $m_2$ rides against a back rest and moves along with $m_1$. Let the masses $m_a = R(1)$ kg and $m_1 = R(2) + R(3)$ kg. If the mass $m_1$ strikes and sticks to the small snubber, how far away from the snubber does block $m_2$ first land. If $m_2$ is treated as a smooth particle how far away from the snubber does it land the second time, if the coefficient of restitution is $e_r = 0.9$ for the box-floor collision. The kinetic coefficient of friction between the blocks is $\mu_k = R(4)/10$. Let $D = 1$ m and $H = 1/2$ m. Let $R(1)$ be the first nonzero digit of your student R-number, $R(2)$ be the second nonzero digit, etc. Thus, if your R-number is $R = 0234607$, then $R(1) = 2$ and $R(2)R(3) = 34$, etc.

\[ V = R(1)R(2) \]

\[ m_a = R(1) \]

\[ m_1 = R(2) + R(3) \]

\[ e_r = 0.9 \]

\[ \mu_k = R(4)/10 \]

\[ D = 1 \]

\[ H = \frac{1}{2} \]

At impact total momentum is conserved and friction forces are

\[ m_1V + m_2V = m_2V_{20} \Rightarrow V_{20} = \frac{(m_1 + m_2)V}{m_2} \]

Block $m_2$ slides along $m_1$ distance $D$ before launching. $s_2 = \mu_k N_2$

\[ \frac{m_2 - m_2g}{m_2} = 0 \rightarrow s_2 = m_2V_{20} \Rightarrow V_s = -\mu_k s \Rightarrow V_{20} = \frac{m_1g}{m_2} \]

\[ \int_{s_2}^{x} \frac{m_1}{m_2} dx = \int_{0}^{s_2} -\mu_k dx \Rightarrow \frac{1}{2} m_1 \frac{dV_2}{dx} - \frac{1}{2} V_2^2 = -\mu_k g (s - s_2) \]

\[ V_2 = \sqrt{V_{20}^2 - 2m_2gD} \]

Block now flies off with horizontal velocity $V_2$

Box in free flight:

\[ x(t) = V_2 t + 0 \]

\[ y(t) = \frac{1}{2} g t^2 + H \]

Box will hit first time when $y(t) = 0$

\[ 0 = \frac{1}{2} g t^2 + H \Rightarrow t_{hit} = \frac{\sqrt{2H}}{g} \]

\[ X_1 = X(t_{hit}) = V_2 \sqrt{2H} \]

\[ X_2 = X(t_{hit}) = V_2 \frac{2\sqrt{2H}}{g} + X_1 \]

\[ X_2 = \frac{1}{2} \left\{ \left[ \frac{(m_1 + m_2)}{m_1} \right] \left[ \frac{(m_1 + m_2)}{m_1 - 2m_2gD} \right] \right\} \frac{2H}{g} \]

\[ X_2 = \frac{1}{2} \left\{ \left[ \frac{(m_1 + m_2)}{m_1} \right] \left[ \frac{(m_1 + m_2)}{m_1 - 2m_2gD} \right] \right\} \frac{8g^2H}{c} + X_1 \]

\[ X_2 = \frac{1}{2} \left\{ \left[ \frac{(m_1 + m_2)}{m_1} \right] \left[ \frac{(m_1 + m_2)}{m_1 - 2m_2gD} \right] \right\} \frac{8g^2H}{c} + X_1 \]
3. (15 pts) The car of weight \( W = R(1)R(3)R(4)R(4) \) lbf is moving at constant downrange speed \( \dot{x} = v \) mph. The hilly road is describes as \( y(x) = a(1 - \cos(2\pi \frac{x}{b})) \) where \( a = R(1) \) ft is the constant amplitude of the hills and \( b = R(2)R(3) \) ft is the period of the hills, as shown. Determine the speed \( v \) that will cause the car to be momentarily weightless at the peaks of the hills. Gravity \( g = 32.17 \frac{ft}{s^2} \) acting down. There are 5280 ft/mile. Let \( R(1) \) be the first nonzero digit of your student R-number, \( R(2) \) be the second nonzero digit, etc. Thus, if your R-number is \( R = 0234607 \), then \( R(1) = 2 \) and \( R(2)R(3) = 34 \), etc.

\[
\begin{align*}
\ddot{y} &= a \left( 1 - \cos \left( \frac{2\pi x}{b} \right) \right) \\
\dot{y} &= \frac{d}{dt} \frac{dy}{dx} = \frac{dy}{dx} \dot{x} = a \frac{2\pi}{b} \sin \left( \frac{2\pi x}{b} \right) \dot{x} \\
\dddot{y} &= \frac{d^2}{dt^2} \frac{dy^2}{dx} = \frac{d}{dt} \left( a \frac{2\pi}{b} \sin \left( \frac{2\pi x}{b} \right) \right) = a \frac{2\pi^2}{b^2} \cos \left( \frac{2\pi x}{b} \right) \dot{x} \\
\end{align*}
\]

At no peak of the hill \( x = \frac{b}{2} \), and forces in \( x \)-direction are zero because of constant \( x \). So \( \dot{N} - W = m \dddot{y} \)

weightless when \( \dot{N} \rightarrow 0 \Rightarrow -mg = m\dddot{y} \Rightarrow \dddot{y} = -g \)

\[
\frac{4\pi^2 a}{b^2} \cos \left( \frac{2\pi x}{b} \right) \dot{x}^2 \bigg|_{x = \frac{b}{2}} = -g \quad \Rightarrow \quad \dot{v} = \sqrt{\frac{g b^2}{4\pi^2 a}} \quad \text{ft} \quad \text{s}^{-1}
\]

\[
V_{mph} = \left( \frac{v}{5280} \right) \left( \frac{3600 \text{ s}}{1 \text{ hr}} \right) \left( \frac{1 \text{ mi}}{5280 \text{ ft}} \right)
\]

\[
V_{mph} = \frac{3600 \sqrt{\frac{g b^2}{4\pi^2 a}}}{5280}
\]

\[
g = 32.17 \frac{ft}{s^2} \\
b = R(2)R(3) \text{ ft} \\
a = R(1) \text{ ft} 
\]
4. (60 pts) Use the six step process. Semaj Dnob is a 700 secret agent, licensed to solve dynamics quandaries. Semaj needs to traverse a height \( H + D \) in order to continue the pursuit of money. Since time is money, develop an expression for the total time it takes for our crooked-smile agent to get from the top to the exit below, develop the expression in terms of \( D, H, g, m_2, \) and \( m_3 \). Assume the conditions are just right that static equilibrium holds for the system before Semaj leaps. Assume our hero is agile enough to not rebound when landing on \( m_2 \), and assume Semaj evenly steps off of \( m_2 \) onto \( m_1 \) when \( D \) first goes to zero. For the contraption shown, the rope passes in front of pulley \( P \) and there is a through hole in the bigger block for \( m_1 \) to pass through. The blocks are apart by a distance \( D \) when it starts to move. Analyze all possible scenarios for the motion of the system. The rope is long enough for the motion to happen for as long as is needed and is strong enough to handle the loads. Ignore friction in the pulleys and air. Gravity acts down. Use the coordinates shown. That is a framer's square he is holding!

1) Jump off - 2 d.o.f
   - Land in \( m_2 \) - 2 d.o.f \( \rightarrow \) 1 d.o.f
   - 1 d.o.f while riding in Box 2
   - 3 d.o.f while stepping from Box 2 onto Box 1
   - 1 d.o.f while on Box 1
   - 3 d.o.f while stepping off - but don't relax at step off

2) coords as shown. \( f'(x, y, z) \) for \( m_3 \)

3) Jump off
   - \( T \)
   - \( m_{3g} \)
   - \( m_{2g} \)
   - \( m_{g} \)
   - \( N \)
   - \( m_{2g} \)
   - \( m_{3g} \)
   - \( m_{g} \)
   - \( N \)
   - \( m_{2g} \)
   - \( m_{3g} \)

while in Box 1
4) Jump off
\[ m_3 \gamma = m_3 \gamma_1 \]
\[ 0 = m_3 \gamma_3 \]
\[ -6T + m_2 \gamma = m_3 \gamma_2 \]
\[ -T + m_1 \gamma = m_1 \gamma_1 \]

While on Box 2
\[ -6T + (m_2 + m_3) \gamma = (m_2 + m_3) \gamma_2 \]

While on Box 1
\[ -T + m_1 \gamma = m_1 \gamma_1 \]

5) \[ L = l_1 + l_2 + l_3 + l_4 + l_5 + l_6 + l_7 + c \]
\[ x_1 = l_1 + c \]
\[ x_2 = l_2 + c \]
\[ x_3 = l_3 + c \]
\[ x_4 = l_4 + c \]
\[ x_5 = l_5 + c \]
\[ x_6 = l_6 + c \]
\[
\text{Step 6)} \quad \text{Jump off (just stay off)}
\]
\[
\begin{align*}
\text{Man} \quad \dot{x}_3 &= 0 \quad \Rightarrow \quad \dot{y} &= g \quad \Rightarrow \quad y(t) &= \frac{1}{2} gt^2 \\
\dot{x}_3 &= x_v' \quad \Rightarrow \quad \text{Time it takes to drop H}
\end{align*}
\]
\[
\begin{align*}
y(H) &= H = \frac{1}{2} g t_H^2 \\
\Rightarrow \quad t_H &= \sqrt{\frac{2H}{g}}
\end{align*}
\]
\[
\text{Boxes}
\]
\[
-6T + m_2 g = m_2 \ddot{x}_2
\]
\[
-6T + m_3 \ddot{x}_1 = m_1 \ddot{x}_1
\]
\[
(m_2 - 6m_1) \dot{g} = m_2 \dddot{x}_2 - 6m_1 \dddot{x}_1
\]
\[
= (m_2 + 36m_1) \dddot{x}_2
\]
\[
\Rightarrow \quad \dddot{x}_2 = \frac{(m_2 - 6m_1) \dddot{g}}{(m_2 + 36m_1)}
\]
So for equilibrium,
\[
m_2 = 6m_1 \Rightarrow m_1 = \frac{1}{6} m_2
\]
\[
\text{Land in } m_2
\]
\[
\text{Landing force is complicated & here in impulse loading in T, so}
\]
\[
\text{Take the system as a whole & look at over a short time of}
\text{landing \Rightarrow system momentum conservation (node negligible at short times)}
\]
\[
\begin{align*}
m_3 \dot{v}_H + m_1 \dot{v}_0 + m_2 \dot{v}_2 &= (m_3 + m_2) \ddot{x}_2 + m_1 \ddot{x}_1
\end{align*}
\]
\[
\begin{align*}
&= (m_2 + m_3 - 6m_1) \dddot{x}_0
\end{align*}
\]
\[
\text{From initial equilibrium}
\]
\[
m_3 = \frac{1}{3} m_2
\]
\[
\text{while in Box 2}
\]
\[
-6T + (m_2 + m_3) \dddot{x}_2 = (m_2 + m_3) \dddot{x}_2
\]
\[
(\dddot{x}_2) = -6 \dddot{x}_2
\]
\[
m_1 = \frac{1}{6} m_2
\]
\[
(m_2 + m_3 - 6m_1) \dddot{g} = (m_2 + m_3 + 36m_1) \dddot{x}_2
\]
\[
\Rightarrow \quad \dddot{x}_2 = \frac{m_2 \dddot{g}}{(m_2 + m_3 + 36m_1)} = A_2
\]
\[
\begin{align*}
\dddot{x}_2(t) &= \frac{1}{3} A_2 t^2 + 7 \frac{1}{2} g H t + H \\
\dddot{x}_1(t) &= -\frac{3}{2} A_1 t^2 - 6 \frac{1}{2} g H t + H + D (\ast)
\end{align*}
\]
\[
\text{Boxes are level when}
\]
\[
\begin{align*}
x_1 &= x_2 \Rightarrow \frac{1}{2} A_2 t^2 + \frac{7}{2} g H t + H = -\frac{1}{2} A_1 t^2 - 6 \frac{1}{2} g H t + H + D
\end{align*}
\]
\[
\frac{1}{3} (A_1 + A_2) t^2 + 5 \frac{1}{2} g H t - D = 0
\]
\[
\frac{7}{2} \frac{m_3 g}{(2m_1 + m_2 + m_3)} \cdot t^2 + 5\sqrt{gH} \cdot t - D = 0
\]

Time of travel to when \( D \to 0 \) and step off occurs

\[
t = - \sqrt{\frac{2gH}{g}} \cdot \left( \frac{7}{2} \frac{m_3 g}{(2m_1 + m_2 + m_3)} \right)^{-0.5} - 5\sqrt{gH}
\]

\[
t = \frac{1}{6} \left( \frac{7m_2 + m_3}{m_2 + m_3} \right) \left( \sqrt{gH} + \frac{14m_3gD}{(13m_2 + m_3)} \right)^{0.5} - 5\sqrt{gH}
\]

Stepping on to box 1

Total momentum again is conserved. Velocities at \( t_D \) are

\[
V_{10} = x_1(t_D) = \frac{6m_3g}{(13m_2 + m_3)} \cdot t_D - 5\sqrt{gH}
\]

\[
V_{20} = x_2(t_D) = \frac{m_2g}{(13m_2 + m_3)} \cdot t_D + \sqrt{gH}
\]

Before

\[
m_1(\dot{V}_{10}) + m_2 \dot{V}_{20} + m_3(\dot{V}_{20}) = m_2v_{2a} + (m_1 + m_3)v_{1a}
\]

\[
\frac{1}{6}m_2v_{10} + (m_1 + m_3)v_{20} = \left( m_2 - \frac{1}{6}m_2 + m_3 \right)v_{2a}
\]

\[
v_{2a} = \frac{\frac{1}{6}m_2v_{10} + (m_1 + m_3)v_{20}}{\left( \frac{35}{36}m_2 - \frac{1}{6}m_3 \right)}
\]

\[
v_{1a} = -6v_{2a}
\]
while on box $k$

$$-6T + m_2 \ddot{y} = m_2 \ddot{x}_2$$
$$-6(-T + (m_1 + m_2) \ddot{y} = (m_1 + m_3) \ddot{x}_1)$$

$$[m_2 - 6(m_1 + m_2)] \ddot{y} = -6(m_1 + m_2) x''_1 + m_2 \dddot{x}_2$$
$$(m_2 - m_2 - 6m_3) \ddot{y} = (36(m_1 + m_2) + m_2) \dddot{x}_2$$

$$x''_2 = \frac{-6 m_3 \ddot{y}}{(36 m_3 + 7m_2)} = -B_2$$

$$x''_1 = \frac{36 m_3 \ddot{y}}{(36 m_3 + 7m_2)} = B_1$$

Need only track $x_1$

$$x_1 (+) = \frac{1}{2} \beta_1 t^2 + 6V_{0a} t + \left( \frac{1}{2} A_1 \left( t_0 \right)^2 - 6 \frac{\sqrt{3} \gamma_1 H(t_0)}{2} (H + D) \right)$$

When $x_1 = H + D$ is when he can stop off.

$$\frac{(H + D)}{2} = \frac{1}{2} \beta_1 t^2 - 6V_{0a} t - \frac{1}{2} A_1 t_0^2 - 6 \frac{\sqrt{3} \gamma_1 H(t_0)}{2} \left( H + D \right)$$

$$t_{H+D} = 6V_{0a} + \left( \frac{36 V_{0a}^2}{4 (1/8)} (\frac{1}{2} A_1 t_0^2 + 6 \frac{\sqrt{3} \gamma_1 H(t_0)}{2} \right)^{1/2}$$

$$t_{H+D} = 6V_{0a} + \left[ 36 V_{0a}^2 + A_1 \beta_1 t_0^2 + 12 \beta_1 \frac{\sqrt{3} \gamma_1 H(t_0)}{2} \right]^{1/2}$$

Total time

$$T_{total} = t_h + t_D + t_{H+D}$$